Orbit Determination Capability Analysis for the Mariner-Jupiter-Saturn 1977 Mission

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Delivering a spacecraft to the outer planets with high precision ($1\sigma \approx 200$ km) and supporting the desired high accuracy instrument pointing near a planetary encounter place great demands on a navigation system. A combined Earth-based radio tracking/onboard optical data navigation system designed to meet these goals, is described in this paper. Some results of applying this navigation system to the preliminary Mariner-Jupiter-Saturn 1977 mission trajectories (the Jupiter approach of the JSX-flight and the Saturn approach of the JST-flight) are presented. It is shown that this system is capable of providing target planet centered orbit determination accuracies in the range of 100 km, and accuracies relative to closely encountered satellites of about 200–250 km, provided there is accurate modeling of the natural satellite motions. Analytical theories of the satellite motions would be attractive from the cost and time usage standpoint as alternatives to numerical integration of the equations of motion. An analytical theory (ignoring the mutual perturbations) is investigated for this role. It is concluded that further development of these theories will have to be undertaken before they can be used for high precision navigation.

Introduction

THE requirements for the high precision delivery of a spacecraft to a close approach to a target body stem, in part, from the desire to maintain the Sun-spacecraft-planet (or natural satellite) geometry to within close proximity of the preflight-planned geometry, so that specific features (e.g., Jupiter's Red Spot, Japetus albedo fluctuations, etc.) can be photographed at optimum illumination. These geometry limits translate into 1σ navigation goals, for the trajectories which are used as examples here, of about 300 km. Requirements for very precise knowledge of the spacecraft's position relative to a target body are the consequences of the desire to accurately point the scientific instruments in the rapidly changing environment near a close encounter. The pointing requirements are designed to ensure that particular objects on the surface of the planet (or satellite) under scrutiny are photographed with the image appearing within a specific quadrant of the picture. These criteria define 1σ navigation goals, for the example trajectories, of about 100 km. These two mission functions levy their requirements on the navigation system at different times during the approach timeline. The information to support the delivery requirement (which must be available prior to the time of the last corrective maneuver) is needed by 4–5 days before planetary encounter (called the "delivery" point), and the information to support the pointing requirements (necessary just prior to the commanding of the detailed terminal sequences) is needed 1–3 days before planetary encounter (called the "knowledge" point).

Knowledge of the position and velocity of the approaching spacecraft with respect to the target body is fundamental to any proposed navigation scheme. With advanced radio tracking techniques, this spacecraft state can be determined to the accuracy of the target planet ephemeris. Onboard optical data are needed to augment the radio data when performances below these levels are desired. This paper will describe the combined radio/optical data navigation system and will show that it will provide the navigation accuracy necessary to meet the delivery and pointing requirements as discussed above.

Description of the Navigation Problem

The process by which the position and velocity of the spacecraft relative to a target are deduced from radio data is illustrated in Fig. 1. The Earth-relative spacecraft position, $\mathbf{X}^E_{S/C}$,

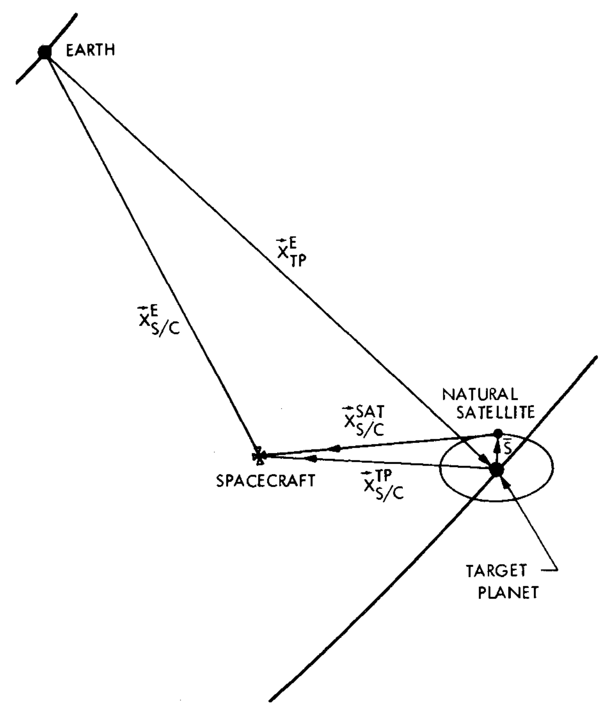

Fig. 1 Earth/target planet and spacecraft geometry.

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Index categories: Navigation, Control, and Guidance Theory; Spacecraft Navigation, Guidance, and Flight-Path Control Systems; Spacecraft Tracking.

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results directly from the reduction of the radio data. If the target were a natural satellite of a planet, it would be necessary to introduce the ephemerides of the Earth and planet relative to the Sun ($\mathbf{X}_{TP}^{E}$), and the ephemeris of the satellite relative to the planet (**S**), to obtain the spacecraft state relative to the target

$$\mathbf{X}_{S/C}^{SAT} = \mathbf{X}_{S/C}^{E} - \mathbf{X}_{TP}^{E} - \mathbf{S} \tag{1}$$

Analyses of radio data orbit determination performance have shown that the position of the spacecraft relative to the Earth can be computed to an accuracy of a few hundred kilometers.[1-5] However, the uncertainties in the ephemerides of the outer planets relative to the Earth are as large as 1500 km,[6] and the uncertainties in the planet relative ephemerides of the natural satellites are approximately the same size. These ephemeris uncertainties limit the performance to be expected from the radio data (especially in far encounter portions of missions) to discouraging levels.[1-5]

An alternative data type, which could diminish the importance of planetary ephemerides in the orbit determination process, would be spacecraft-centered inertially referenced planet direction measurements using optical sensing devices. There are several suggested forms of these measurements using the science TV camera, or additional optical sensors to be mounted onboard the spacecraft. The MJS'77 mission will use measurements from TV photographs of the natural satellites of a target planet against star backgrounds for its spacecraft-centered measurements. The detailed reasons for this choice are discussed in Ref. 17.

Observations of the satellites against the stars measure directly two components of the unit vector along $\mathbf{X}_{S/C}^{SAT}$ (see Fig. 1) relative to an inertial coordinate system. A succession of these measurements, as the natural satellite revolves around its primary, will allow the accurate determination of the target-planet-centered satellite ephemeris (**S**), and then the target-planet-centered spacecraft position ($\mathbf{X}_{S/C}^{TP}$) can be computed from

$$\mathbf{X}_{S/C}^{TP} = \mathbf{X}_{S/C}^{SAT} + \mathbf{S} \tag{2}$$

It is noted that a single star/satellite observation does not measure the line-of-sight distance to the satellite being viewed. This characteristic of the data type (illustrated in detail in Ref. 17) is a serious drawback to using the optical data by itself for navigation, as it impedes the determination of the distance along the trajectory from the epoch of a data arc to planetary encounter for the spacecraft. However, it is noted[1-5] that the determination of the planet-to-spacecraft range using radio data is the least compromised by the large uncertainties on the target planet ephemeris. Hence, combining the radio data with the onboard optical data in a single navigation system should give significantly better solutions for the target-planet-centered spacecraft state than any derivable from either data type separately.

Description of Data Types and Error Sources

The radio data types expected to be available for navigating the MJS'77 missions are range and range rate (Doppler). These data have several inherent sources of error in addition to the ephemeris problem associated with their use. These error sources include random data noise, Earth-bound station location errors, transmission media errors, Earth polar motion uncertainties, two and three way range and range rate biases, and Earth spin rate errors. The transmission media errors, Earth polar motion errors, data biases and Earth spin rate errors are all subject to calibration efforts. However, these calibrations are not exact, and it is assumed, for the studies to support this paper, that the residual errors from the calibration efforts can be represented adequately by considering them as contributors to the constant station location errors.

There are two additional problems associated with the application of these data to the orbit determination process: 1) the corruptive effects of small, unmodeled, nongravitational forces acting on the spacecraft (e.g., leaking attitude control jet valves, etc.); and 2) poor determinability of the Earth-centered declination whenever the spacecraft is near 0° declination, relative to the Earth's equator plane. However, it has been shown[5-7] that whenever the spacecraft is being tracked simultaneously by two stations, these two problems can be averted by differencing the data sets from the two stations prior to processing. These quasi-interferometric data will be used for the orbit determination processes for the MJS'77 missions.

Table 1 Assumed uncertainties values on model parameters for navigation analysis model (3σ)

Radio error source[5]	Magnitude (3σ)
1) Station locations	
a) Distance off spin axis	3 m
b) Longitude	6 m
c) Distance off equator	15 m
2) Spacecraft nongravitational acceleration	
a) Time varying	3×10^{-12} km/sec^2
b) Constant	3×10^{-12} km/sec^2
3) Data jitter	
a) Doppler, *S*-band, and *X*-band, 2-way, 3-way	3 mm/sec over 1 min count time
b) Range	9 m
Optical error source[13]	**Magnitude (3σ)**
1) Data noise	90 μrad
2) Biases	90 μrad

The uncertainties in the station locations, nongravitational accelerations (which are represented for study purposes by a time-correlated stochastic process), and data noise that were used for these studies are listed in Table 1. It is assumed that these numbers describe the radio tracking characteristics in the mission time interval.

The optical data will consist of photographs of the natural satellites of the target planets against star backgrounds. The data, for these studies, will be taken with a TV camera similar to the narrow-angle camera on the Mariner IX[8] spacecraft. The attitude control sensors and the scan platform arrangement will also be similar to those on Mariner IX.

The main error sources in the optical data are biases and distortions in the spacecraft-scan platform-TV camera system. The biases can take several forms: there could be biases in the attitude control system, biases in the scan platform pointing, electromagnetic effects in the TV system due to the workings of the TV system, electromagnetic effects due to the presence of the other onboard instruments, or optical effects due to defects in the manufacture or the mounting of the lenses.[10] Systematic distortions can arise from repeatable electromagnetic effects, or optical effects in the lenses. The biases and distortions are subject to preflight and inflight calibration schemes, such as those discussed in Ref. 11. These calibrations are not exact; therefore, it is assumed that the biases and distortions are calibrated to such a level that the remaining errors are bounded by the noise level. In each study, a bias uncertainty, equal in size to the noise level, has been taken into account. The assumed noise and bias levels are also listed in Table 1.

In general, since the natural satellites will be much brighter than the stars in the photographs, the images of the satellites will be overexposed to detect the stars. Overexposure will cause the satellite images to "bloom," or spread out on the vidicon facade. For these studies it is assumed that the "blooming" effects are symmetrical, thereby allowing the center of the image to be located with the same accuracy as a properly exposed image. Laboratory tests and Mariner IX experience have shown that this assumption is a reasonably accurate description of reality.[9]

The taking and processing of such pictures for navigation purposes was demonstrated to be feasible during the Mariner IX mission.[9] Three sequences, totaling 21 photographs of Phobos and Deimos, were planned and executed, and the resulting data

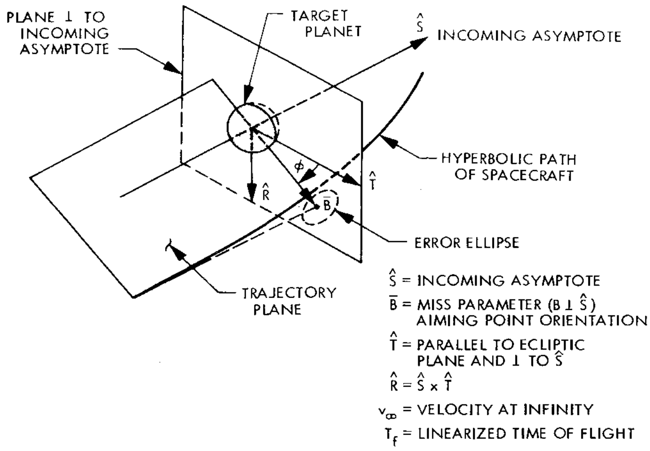

Fig. 2 *B*-plane coordinate system.

were included in the orbit determination process. The Mariner IX data were corrupted by 15 μrad noise, 1σ. These navigation accuracy studies assume a more pessimistic data quality than the Mariner IX results would dictate, as an attempt has been made to account for camera degradations, which could possibly be caused by the extended length of the outer planet missions, and data degradation due to dynamic range difficulties with the camera.

Description of the Preliminary MJS'77 Mission

As representative of the large set of possible multiouter planet missions, two trajectories have been selected for preliminary mission design studies. The design details of these trajectories (commonly called the JST- and JSX-flights) are described by Bourke et al.[12] and the trajectory characteristics are described in Ref. 16. The JSX-flight will be launched in the fourth quarter of 1977, with the JST-flight being launched about 15 days later. About 1.6 yr later, the two spacecraft will encounter Jupiter, with the JST-flight encountering about one and a half months before the JSX-flight. Fifteen and one-half hr after the JSX-flight Jupiter encounter, it makes a 17,000 km close approach to Ganymede, going directly under the satellite. Both spacecraft will continue on to Saturn and encounter it about 1.8–2.0 yr after departing Jupiter. The JST-flight (which has a 10,000 km close approach to Titan $19\frac{1}{2}$ hr before Saturn encounter) arrives at Saturn about three months ahead of the JSX-flight. The Jupiter approach of the JSX-flight and the Saturn approach of the JST-flight are used as the reference trajectories for the studies to support this paper.

Application of Navigation Scheme to Representative Missions

The results of using the radio and optical data for orbit determination will be discussed in both planet-centered and natural satellite-centered reference frames. For planet-centered discussions, the *B*-plane coordinate system, which is shown in Fig. 2, will be used. The uncertainties on the determination of the spacecraft position and velocity will be quoted as the semi-major and semiminor axes (SMAA and SMIA, respectively,) of the projection of the error ellipsoid in the *R–T* plane, and the length of the projection of that error ellipsoid along the *S*-axis divided by the planet relative velocity at an infinite distance away (time of flight). For satellite centered results, the "viewing" coordinate frame (Fig. 3) at the time of spacecraft satellite encounter will be used.[13] In this frame, the uncertainty ellipse in the plane perpendicular to the *V*-axis (target plane) contains the uncertainty in the determination of the spacecraft-to-satellite distance, while the uncertainty ellipse in the plane perpendicular to the *R*-axis (viewing plane) determines pointing accuracies for the scientific instruments.

The small, nongravitational accelerations, which were detrimental to navigation estimates using only the conventional radio data, were investigated as a possible menace to the combined radio/optical data navigation system. In a study using quasi-interferometric radio data and optical data from two satellites, Titan and Rhea, of Saturn (with the optical data arc extending from encounter minus thirty days to the "delivery" point) constant three-axis nongravitational acceleration of 10^{-12} km/sec^2 (1σ) were considered along with the errors discussed above. The differences between the solutions considering accelerations and those where accelerations were not present were only 0.2 km in semimajor axis of the *B*-plane uncertainty ellipse, 0.05 km in the semiminor axis, and 0.11 sec ($\approx$ 1.5 km) in time of flight. These results tend to imply that the non-gravitational accelerations, stochastic or constant, are not significant error sources for the combined radio/optical navigation solutions.

The lack of knowledge of the target planet ephemeris, which dominates the radio data navigation performance estimates, is also shown not to be a significant contributor to the navigation uncertainties when the combined radio/onboard optical data navigation system is used. In a study using the same data pattern and arc as the unmodelled acceleration investigation the Brouwer

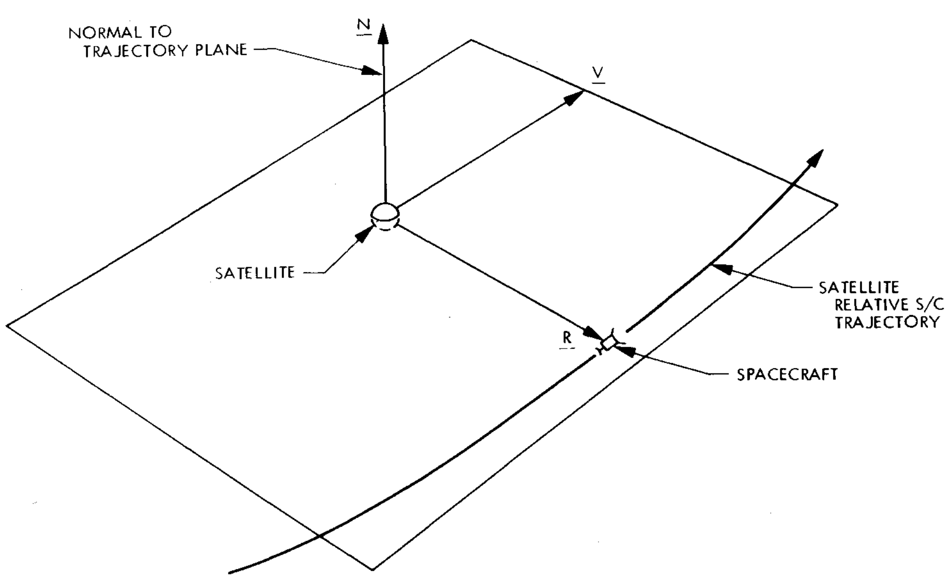

Fig. 3 Viewing coordinates.

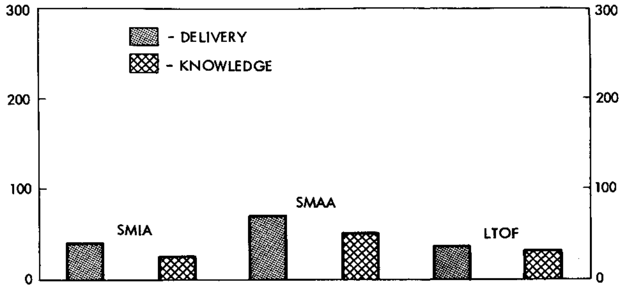

Fig. 4 Jupiter-centered B-plane uncertainties.

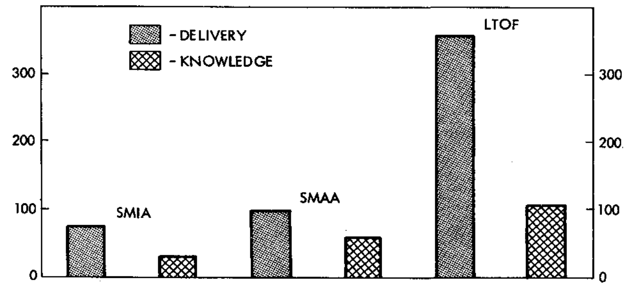

Fig. 5 Saturn-centered B-plane uncertainties.

Table 2 Saturn ephemeris uncertainties before and after combined Earth-based radio/onboard optical data navigation solution

Parameter	Before solution	After solution[a]
$\Delta a/a$	0.6×10^{-7}	0.6×10^{-7}
Δe	0.3×10^{-6}	0.298×10^{-6}
$\Delta M_0+\Delta W$	0.6×10^{-6}	0.598×10^{-6}
ΔP	0.3×10^{-6}	0.3×10^{-6}
ΔQ	0.3×10^{-6}	0.3×10^{-6}
$e\Delta W$	0.3×10^{-6}	0.3×10^{-6}

[a] At Saturn "delivery" point (encounter −4 days).

and Clemence set III parameters[14] for the ephemerides of Saturn were included in the list of solution parameters. It is seen in Table 2 that the uncertainties in Saturn's ephemeris elements are virtually unchanged by the addition of optical data to the orbit determination process. This indicates that the combined radio/optical data navigation estimates are practically insensitive to the presence of ephemeris uncertainties, as was expected from the previous theoretical discussion. The relevant B-plane uncertainty ellipse parameters are only slightly affected by these errors: the semimajor axes of the B-plane uncertainty ellipses differ by 0.007 km, as do the semiminor axes, and the times of flight differ by 0.002 sec ($\approx$0.3 km).

The combined radio/optical data navigation system was used on the MJS'77 mission trajectories (JSX-flight at Jupiter and JST-flight at Saturn) in a study to determine the expected orbit determination performance. Optical data from the four Galilean satellites at Jupiter (at the rate of one picture of each satellite every four hours on a data arc of encounter minus thirty days to encounter) and from two satellites at Saturn (Rhea and Titan with similar data rates and data arc) were used with the quasi-interferometric radio data for these studies. The results are shown in Fig. 4, for the Jupiter "delivery" and "knowledge" points, and Fig. 5, for the Saturn "delivery" and "knowledge" points. It is noted that, at the "delivery" points, the semimajor axes of the uncertainty ellipses are about 70 km (Jupiter) and 100 km (Saturn), the semiminor axes are about 40 km (Jupiter) and 75 km (Saturn), and the time of flight uncertainties are about 35 km (Jupiter) and 350 km (Saturn). The primary reason for the better results at the Jupiter encounter is that more optical data were included in the study than in the Saturn study.

Knowledge of the planet-centered ephemerides of the natural satellites is important for accurate pointing of the scientific instruments at satellites other than the encountered ones. Studies were made using optical data from Io, Europa, Ganymede and Callisto at Jupiter, and Tethys, Rhea, Titan, Hyperion, and Japetus at Saturn (with one picture of each satellite every 4 hr on data arcs extending from encounter minus thirty days to encounter minus two days), and the quasi-interferometric radio data. The results, shown in Fig. 6 (for Jupiter) and Fig. 7 (for Saturn), are quoted as uncertainties in the radial distance from the planet (σ_w), along the orbit path (σ_v), and perpendicular to the orbit path (σ_u). It is seen that ephemeris accuracies of less than 100 km are expected at Jupiter, and accuracies in the 100 km–150 km range are expected at Saturn. Better accuracies in the

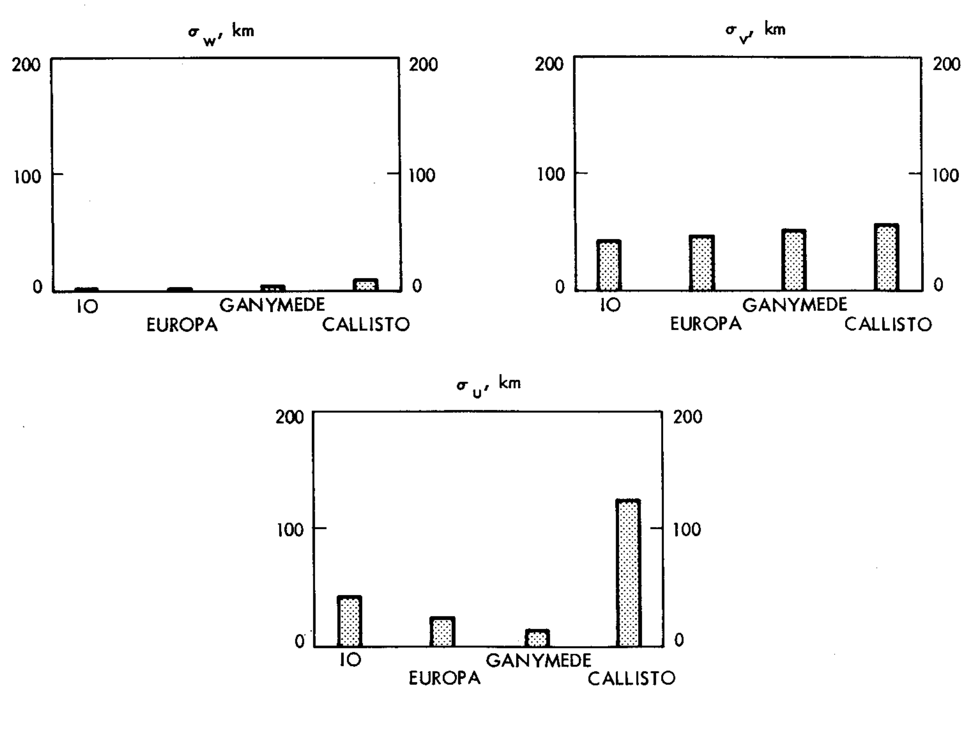

Fig. 6 Jupiter-centered satellite ephemeris uncertainties at "knowledge" point.

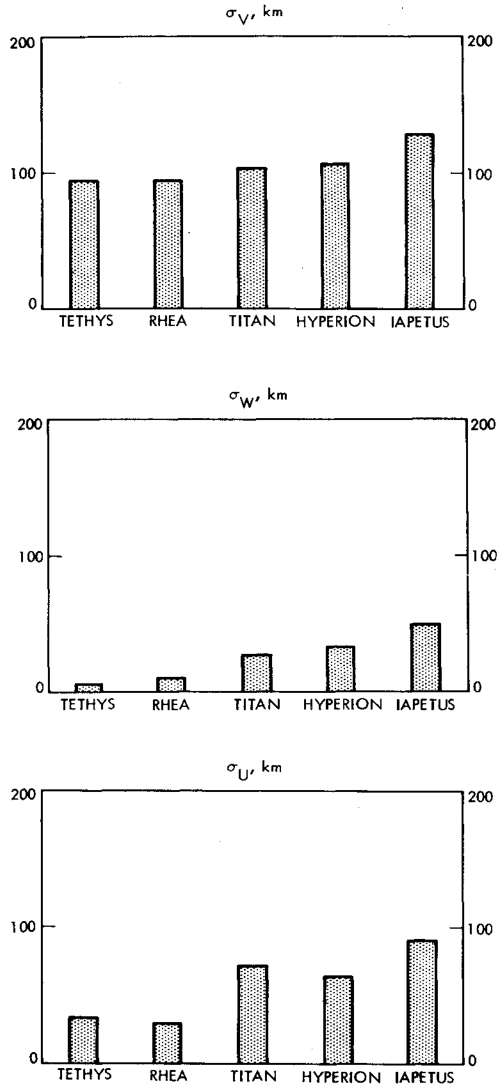

Fig. 7 Saturn-centered satellite ephemeris uncertainties at "knowledge" point.

ephemeris determination are expected at Jupiter because shorter period satellites are used for the optical data at Jupiter. It has been shown[13] that spreading a given number of pictures over many revolutions of a satellite, in general, gives better results for satellite ephemeris determination than taking the pictures within a single revolution of the satellite.

The MJS'77 trajectories are targeted for close encounters to two satellites (JSX-flight 17,000 km approach to Ganymede at Jupiter and JST-flight 10,000 km approach to Titan at Saturn). For this reason, the "delivery" and "knowledge" accuracies with respect to Ganymede and Titan are as important as these accuracies with respect to Jupiter and Saturn. These satellite relative uncertainties control the accuracy with which the projected close flybys can be targeted, and the accuracy with which the scientific instruments can be pointed at the satellites. The results of using the combined radio/optical data navigation system are shown in the target planes of Ganymede and Titan in Fig. 8 and in the viewing planes of Ganymede and Titan in Fig. 9. It is seen that the satellite relative accuracies are in concert with the high precision navigation requirements (approximately 300 km for "delivery" and approximately 100 km for "knowledge").

The navigation accuracy studies that are reported here are incomplete in that they have treated two major problem areas in a nonrigorous fashion. Assumptions have been made concerning the symmetry of blooming and the subsequent ability to locate

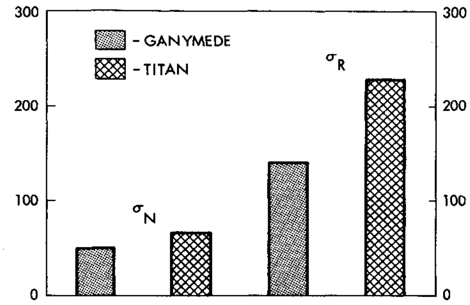

Fig. 8 Target plane uncertainties at planet encounter minus four days.

the center of the satellite image; and it has been assumed that the results derived using a model for the satellite dynamics which ignores the satellite mutual perturbations are characteristic of the results to be derived using a more complete satellite dynamics model. This last assumption has been justified, to some degree, by the argument that the parameters characterizing the perturbing effects can be added to the list of solution parameters if a more complete dynamical model is used, and estimated along with the spacecraft trajectory and viewed satellite ephemeris. The navigation effects of ignoring the mutual perturbations will be investigated in more detail in the next section.

Satellite Dynamics Mismodeling Effects

The motions of the natural satellites must be described to allow the optical data to determine the spacecraft position and velocity relative to the target planet. The character of these motions must also be accurately modeled so that updates to the natural satellite orbit description parameters can be derived from the data. Three methods exist to date for describing satellite motions: 1) numerical integration of the satellite equations of motion, 2) a literal theory of the satellite motions, and 3) a time series expansion of certain orbit description parameters, where the values for the coefficients are derived by fitting the motions to Earth telescopic observations. Each of these alternatives has drawbacks. Numerical integration techniques are costly, time-consuming procedures for satellites with periods as short as a day. Time series expansions are undesirable because the partial derivatives necessary to update the parameters of such a theory are unavailable. Literal theories would be less expensive to use than numerical integration, but a literal theory of the motion of a satellite which is experiencing both the attractions of a central body and perturbations from satellites in its own planetary system has not been formulated.

An investigation involving the simulation of optical data from a time series expansion of Sampson's tables (which include some mutual perturbation effects) and fitting the data with a state-of-the-art literal theory (Aksnes' theory,[15] which does not include mutual perturbation effects), was conducted to obtain a preliminary estimate of orbit determination errors that would result from modeling the satellite dynamics with a literal theory.

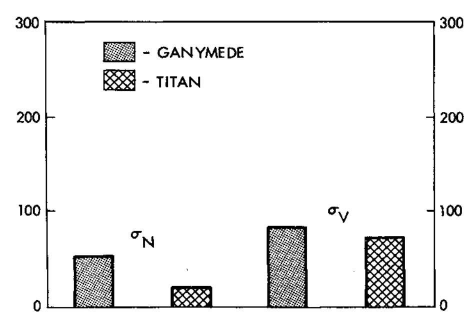

Fig. 9 Viewing plane uncertainties at planet encounter minus two days.

The details of this study are described in Ref. 17. By comparing the sizes of the corrections to the *B*-plane parameters with the uncertainties of these parameters, such a preliminary evaluation of the mismodeling effects can be made. It was seen that 3000 km–12,000 km errors in modeling the satellite dynamics manifested themselves as 4σ–5σ (500 km) navigation errors. While this study hardly qualifies as a Monte Carlo simulation, as simulated data studies should, it does provide some insight into the magnitudes of the mismodeling problem. It is noted that an analytical theory, which offers much in the way of speed and savings in cost, would have to be developed further than the present state-of-the-art to be acceptable for navigation usage. The numerical integration alternative, while costly and time-consuming, is accurate enough to provide the needed dynamical descriptions, but a more complete analytical theory would be highly desirable for future savings.

Conclusions

It has been shown that under the ideal conditions described previously the combined radio/optical data navigation system can support high precision missions to the outer planets. Planet-relative spacecraft trajectory determination accuracies of 100 km, or less (except time of flight at Saturn "delivery") are expected at both the "knowledge" and "delivery" points. Targeting relative to the natural satellites is expected to be accomplished to accuracies of 250 km or less. The high precision pointing requirements, with respect to the encountered satellites, are expected to be met with position uncertainties at the "knowledge" point of approximately 100 km. The planet relative satellite ephemerides, for pointing the camera at other satellites, are expected to be known to less than 150 km, at the "knowledge" point. Finally, while the speed and lower costs of use make analytical theories of the satellite dynamics attractive, the navigation errors associated with using such theories in systems where there are significant satellite-satellite perturbations make their use untenable for high precision navigation.

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